Uncertainties Assesment For Safety Margins Evaluation In Mtr Reactors Core Thermal-Hydraulic Design

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ABSTRACT

This report contains a bibliographic review and a critical analysis of different methodologies used for uncertainty evaluation in research reactors core safety related parameters. Different parameters where uncertainties are considered are also presented and discussed, as well as their intrinsic nature regarding the way their uncertainty combination must be done. Finally a combined statistical method with direct propagation of uncertainties and a set of basic parameters as wall and ONB temperatures, CHF, PRD and their respective ratios where uncertainties should be considered is proposed.

Critical Review of Different Methodologies for Uncertainties Treatment

When a reactor is in the design stage, it should not be presumed that, when constructed, it would be an exact copy of the calculations. Deviations, uncertainties from the fabrication and construction processes, unknowns and simplifications made in the thermal and neutronic analysis should be taken into account as well as possible deviations in the operational conditions. Safety margins are introduced to give the needed safe flexibility in operating conditions. These uncertainties must be taken into account when these margins are evaluated. Some reactor designs have large safety margins, and large uncertainties can be assumed without any particular difficulty. Even in these cases the choice of overly conservative peaking factors can unnecessarily limit the range and usefulness of the reactor [1]. Experience will help reduce the uncertainty, but conservatism should increase with the degree of lack of knowledge [2].

The way uncertainties are taken into account and the nature and magnitude of these quantities are dependent on the model chosen, the degree of sophistication of the analysis, the amount of experimental work to support and correct the analysis and the type of reactor under consideration [3]

In some textbooks, uncertainties affecting temperature rise, power generation or heat flux, are usually estimated and stated in terms of hot-spot and hot channel factors when safety margins are evaluated. These factors are the ratio of the maximum expected values, which take into account uncertainties, to the nominal average values. These factors include contribution from several basic quantities or parameters, x_i which have their own uncertainty, Δx_i and mean value $\bar{x}_i$, defined as $x_i = \bar{x}_i \pm \Delta x_i$. These uncertainties may be expressed as basic sub factors:

$$f_i = 1 + \frac{\Delta x_i}{\bar{x}_i} \qquad (1)$$

A distinction between engineering and nuclear sub factors is used and it is a convenient way to indicate the origin of the uncertainty. The nuclear sub factors are concerned not only for inaccurate determination of the neutron fluxes but also for the spatial distribution of the volumetric heat sources. As a result the nuclear sub factors are much larger than the engineering sub factors.

This methodology based on factors is actually a simplified bookkeeping method of taking care of this complex situation, as it fails to account for the interaction between separate factors [3]. Other methodologies as error propagation tend to solve this limitation.

An accurate method to evaluate the different parameter uncertainties should take into consideration that some are deterministic in nature and others are stochastic in the sense that a given value of these parameters has a certain probability of occurrence at a given location and a given time [3].

It is important to distinguish between variables subject to random-type uncertainties and those introduced to account for inadequate knowledge. The random uncertainties derived primarily from manufacturing tolerance experience lend themselves to statistical analysis. It is not valid however, to attempt statistical treatment of factors that describe uncertainties in performance. Experienced designers can some times established confidence limits by subjective judgment and then use statistical procedures [4].

Engineering Uncertainties

An extensive list of basic parameters where uncertainties are expected was found in the bibliography as well as the way they are considered by the different authors. Those related with correlations used in different calculus are summarised in Table 1. Uncertainties in heat transfer, CHF, ONB and PRD correlations appear as consequence of stochastic errors in the measurements of experimental data used to develop a given correlation. But also a systematic error should be considered when using a given correlation to predict a particular phenomenon in a particular core, because it might have been derived from different geometry or conditions.

Furthermore the over or under estimation, that is the correlation uncertainty, will affect all the channels in the same way, and for this reason should be combined systematically. Therefore a statistical combination of this uncertainty with those coming from fabrication tolerances will not be legitimate. On the other hand the statistical uncertainties of the parameters used to evaluate the correlation should be statistically combined to evaluate wall temperature, CHF, ONB and PDR.

Correlations uncertainties		
Parameter	Uncertainty treatment	Reference
CHF correlation	Statistic	Yoder, ANS, [5]
FE correlation	Statistic	Yoder, ANS, [5]
Initiated Boiling correlation	Statistic	Yoder, ANS, [5]
Forced convection heat transf. corr.	Statistic	Yoder, ANS, [5]
Forced convection heat transf. corr.	Systematic	Woodruff, Georgia-ANL[6]
Forced convection heat transf. corr.	Systematic	Woodruff, ANL [1]
Forced convection heat transf. corr.	Deterministic in nature	Fenech, Rohsenow, MIT [3]
Heat transfer correlation	Systematic	Mishima, KURRI [12]

Table 1.- Engineering uncertainties related with correlations

In Table 2 parameters related with fabrication uncertainties are summarised. Fabrication uncertainties are statistical by nature and as it is assumed that tolerances will not occur at the same position, it is more realistic to consider a statistical combination of them. Moreover of manufacturing tolerances that affect coolant channel gap, uncertainties due to operational environment conditions should be considered depending on plate temperature variation, pressure differences across the plates and fuel swelling during burn-up, among others.

Uranium loading represents the average fuel loading along the fuel plate. In ANS calculation [5], uranium loading uncertainties are named as integral hot streak and are taken into account for liquid bulk temperature increase.

Fuel density or homogeneity accounts for local variations of uranium and is measured in a certain area and compared with fuel average density. Meat thickens and fuel density, are considered to evaluate local effects in heat flux due to tolerances. Meat thickness and/or uranium density variations in ANS report are named as hot streak fuel segregation uncertainties.

In Table 3, parameters related with calculation uncertainties are summarised.

Fabrication uncertainties		
Parameter	Uncertainty treatment, nature of parameters & comments	Reference
U density or homogeneity	Statistic: Affects liquid temperature and heat transfer coefficient	Woodruff, Georgia-ANL [6]
U homogeneity	Statistic. Affects q" (local) and liquid temperature (average along plate)	Woodruff, ANL [1]
U density	Statistic: Affects q" to consider local effects	Mishima, KURRI [12]
Hot streak fuel segregation	Local fuel loading variations Deterministic: affects wall temp. and q"	Yoder, ANS, [5]
U enrich. & density	Statistic	Fenech, Rohsenow, MIT[3]
Fuel loading/plate	Statistical: Affects liquid temp. and q"	Mishima, KURRI [12]
U loading	Statistic: Affects q" and liquid temperature	Woodruff, ANL [1]
U loading	Statistic: Affects q" and liquid temperature	Woodruff,Georgia-ANL [6]
Integral hot streak	Avg fuel loading variations along fuel plate Statistic: affects liquid temperature	Yoder, ANS, [5]
Fuel Meat thickness	Statistic: affects q"	Woodruff, Georgia-ANL[6]
Fuel Meat thickness	Statistic: affects q"	Woodruff, Georgia-ANL[6]
Fuel thickness	Statistic: affects q"	Mishima, KURRI [12]
Fuel heated length	Statistic: affects liquid temperature	Yoder, ANS, [5]
Cool. channel gap	Statistic: manufact. & operat. environment	Yoder, ANS, [5]
Cool. channel gap	Statistic: affects liquid temperature and heat transfer coefficient.	Woodruff, Georgia-ANL[6]
Channel thickness/ hydraulic diameter	Statistic: affects liquid temperature and heat transfer coefficient	Mishima, KURRI [12]

Coolant channel spacing	Statistic: affects liquid temperature and heat transfer coefficient	Woodruff, ANL [1]
Manufacture errors	Statistic	Fenech, Rohsenow, MIT[3]

Table 2 - Engineering uncertainties related with fuel and channel

Computation uncertainties		
Parameter	Uncertainty treatment or nature of the parameter & comments	Reference
Streak-average power density distribution	Statistic	Yoder, ANS, [5]
Local power density distrib.	Statistic	Yoder, ANS, [5]
Computed power density	Statistic	Woodruff, ANL [1]
Calculated power density	Statistic	Woodruff, Georgia-ANL[6]
Power density calculations	Systematic: Affects liquid temp. and q”	Mishima, KURRI [12]

Table 3 - Uncertainties related with calculus

Parameters related with operational control and measurements uncertainties are summarised in Table 4. Fluctuations in measurement or in control operations could be considered as statistical by nature, but as these fluctuations affect the entire core, they should be combined systematically with other uncertainties. Therefore a statistical combination of this uncertainty with those, for example, coming from fabrication tolerances is not adequate.

Measurement (operational) uncertainties		
Parameter	Uncertainty treatment or nature of the parameter & comments	Reference
Reactor power	Statistic: control margin uncertainties (2σ) and measurement errors (3σ)	Yoder, ANS, [5]
Power level measurement	Assumes 95% of full power deposited in fuel Statistic: Affects liquid temperature and q”	Woodruff, ANL [1]
Power measurement	Systematic: Affects liquid temperature and q”	Mishima, KURRI [12]
Power measurement	Statistic: Affects liquid temperature and q”	Woodruff, ANL [1]
Power fluctuations errors operating conditions	Affects liquid temperature and q”	Fenech, Rohsenow, MIT [3]
Inlet coolant temp	Worst case: control margin uncertainties (2σ) and measurement errors (3σ)	Yoder, ANS, [5]
Inlet coolant temp	Affects Fb and Fh	Fenech and Rohsenow, MIT [3]
Flow	Worst case: Control margin uncertainties (2σ) and measurement errors (3σ)	Yoder, ANS, [5]
Coolant flow rate	Statistic: Affects liquid temperature and heat transfer coefficient	Woodruff, Georgia-ANL [6]

Flow measurement	Systematic: Affects liquid temperature and heat transfer coefficient	Mishima, KURRI [12]
Coolant flow rate, distribution	Deterministic in nature or statistical	Fenech, Rohsenow, MIT [3]
Coolant velocity	Systematic: Affects liquid temperature and heat transfer coefficient	Mishima, KURRI [12]

Table 4 - Engineering uncertainties related with operational measurements

No uncertainties were considered for variables such as: water properties, atmospheric pressure and water level**.**

Methodologies to combine uncertainties

The merits of diverse methods to perform uncertainty analysis, both statistical and nonstatistical, are discussed in what follows.

Worst Case Approach: In this approach a conservative constant value is assigned to each parameter reflecting the maximum deviation from nominal value (i.e., typically 2 or 3 standard deviations from the mean). All uncertainties are assumed to exist simultaneously at their maximum values. This method was common in the past. However, this approach was judged to be unnecessarily conservative, and it does not provide any quantitative measure of the level of risk involved. The primary advantage of the approach is the conceptual simplicity because detailed uncertainty distributions for the input parameters and correlations are not required.

Statistical Methods: Holding that the probability of the extreme values of the deviations, all occurring at the same point in the core is quite remote, the multiplicative approach is replaced by a more realistic one in which the chosen standard deviations are combined in a statistical manner [2]. These methods of combining the engineering uncertainties are more accurate but fewer conservatives than the multiplicative or worst case method, and take into account the statistical nature of most of the engineering subfactors [3]

Several different approaches to the statistical analysis have been used depending on the amount of information available on the statistical behavior of the variables and the type of reactor being analyzed.[3]. The statistical approach was first suggested by Tourneau and Grimble [**8**] described in some detail by Hitchcock [9] and first used in an actual design on the Erico Fermi fast-breeder reactor [10].

It is reasonable to assume that deviations from a mean value are random and hence follow a normal, or Gaussian, distribution [**4**]. The standard deviation is a measure of the dispersion of the data. When a given variation may be caused by several independent factors, the total variance can be obtained from the sum of the squares of the individual standard deviations. Uncertainties that are not related (correlated) to one another can also be combined through the use of the standard deviations. If X is an arbitrary function of independent random variables, $x_1, x_2, x_3, \dots, x_n$, with a mean value, $\overline{X}$, and considering the first order terms of the respective Taylor expansion, the following expression is obtained,

$$\Delta X = \frac{\partial X}{\partial x_1}\Delta x_1 + \frac{\partial X}{\partial x_2}\Delta x_2 + \ldots + \frac{\partial X}{\partial x_n}\Delta x_n \tag{2}$$

And the normal variance, σ^2, of such a function can be expressed as, [11] and [3]

$$\sigma_f^2 = \sum_{i=1}^{n}\left(\overline{\frac{\partial X}{\partial x_i}}\right)^2 \sigma_{x_i}^2 \tag{3}$$

Where $\partial X/\partial x_i$ is the partial derivative of X with respect to x_i at the nominal conditions.

Finally when applying the statistical method, it is first necessary to decide upon the number of standard deviations in order to keep a degree of conservatism in the design of the reactor. As a general rule it is taken to be 3.

Weighted Or Combined Statistical Method: When the error for variables is not statistical but systematic the above fully statistical combination method may be not applicable [12]. If X is an arbitrary function of parameters x_i , and the error for $x_1,\ldots,x_M$ is statistical and that for $x_{M+1},\ldots,x_N$ is not, a mixed combination method can be used. When using a channel factor technique the overall factor, F, is evaluated as the product of the systematic factors and the statistical factor in the following way:

$$F = \prod_{i=M+1}^{N}\tilde{f}_i \left(1 + \sqrt{\sum_{i=1}^{M}(\tilde{f}_i - 1)^2} \right) \tag{4}$$

Where $\tilde{f}_i$ is named as derived subfactor, and is expressed as:

$$\tilde{f}_i = \frac{X(x_i + \Delta x_i)}{\bar{X}} \tag{5}$$

The above procedure was used by Chelemer and Tong [18] for open-lattice pressurized-water reactors, and Mishima [12] for research reactors.

Monte Carlo Method: Another statistical approach is combining input uncertainties randomly (through sampling of individual uncertainty distribution) in successive simulations. The most common technique is Monte Carlo method. The disadvantages include the need to provide detailed errors distributions for each of the parameters and correlations. This affects appreciably the reactor performance and the computer time that might be involved in performing the many statistical sampling combinations. There are variations of this methodology like the Latin hypercube sampling that improve the efficiency of the method.

Table 5 summarises consulted reports and textbooks with the methodologies used or proposed in each one.

References	Methodology to combine uncertainties
Thermal-Hydraulic analysis for core conversion to the use of low-enriched uranium fuels in the KUR, K. Mishima, K Kanda and T. Shibata, Kurri-tr-258, 1984 [**12**]	Mixed combination
Steady-state Thermal-hydraulic design analysis of the Advanced Neutron Source Reactor (Cap 4), G. L. Yoder, Jr, ANS. [5]	Monte Carlo & worst case
Evaluation and Selection of Hot Channel (peaking) Factors for Research Reactor Applications. Woodruff, ANL. [1]	Statistical combination
Analyses for Conversion of the Georgia Tech Research Reactor from HEU to LEU fuel. J. E. Matos, S.C. Mo. and W.L. Woodruff [6]	Statistical combination
Transient Analyses and Thermal hydraulic safety margins for the Greek Research Reactor (GRR1), W.L. Woodruff and J.R. Deen.[13]	Statistical combination
Enrico Fermi Reactor, Tables of Factors or One-Sided Tolerance Limits for a Normal Distribution, Report SCR-13, Sandia Corp., 1958. [14]	Statistical Method
Heat Transfer (Chapter 16). Fenech H. and Rohsenow W.M. MIT 1973[3]	-Conventional method: multiplicative -Statistical Method -Weighted statistical (mixed combination)
Nuclear Heat Transport, M. M. El-Wakil (Text book)[2]	-Multiplicative approach -Statistical approach
Thermal Analysis of pressurised water reactors, L. S. Tong&J. Wiesman (Text book) [15]	-Multiplicative approach -Statistical evaluation of fabrication tolerance subfactor
Elements of Nuclear Reactor Design, Weisman, (Text book) [16]	-Multiplicative approach -Statistical approach: for fabrication tolerances evaluation
Nuclear Power Plant, M. Cumo, N Afgan, (Text book) [17]	-Product method -Statistical method: more realistic design for variables which are statistical in nature

Table 5 - Methodologies for uncertainties evaluation

Finally it is important to remember that although large hot-channel or engineering factors are desirable for safety and reliability, it is preferable to use factors that will give only a reasonable degree of confidence that the design limits will not be exceeded. Otherwise, core performance will be penalized [**4**].

Methodology Developed For Uncertainties Treatment In Core Design

Based on the previous analysis, a weighted statistical method with direct error propagation and a set of parameters that present uncertainties is proposed to evaluate safety margins for experimental pool type reactors core design. The meaning of direct error propagation is that there is no approximation in the error evaluation like in the methodology where the first order terms of the Taylor expansion of the function is considered.

Adopted Methodology For Uncertainties Propagation

A mixed or combined methodology based on direct uncertainty propagation is adopted. Considering X, a safety-related variable as an arbitrary function of variables x_i, it can be expressed that:

$$X = \overline{X} \pm E(X) \tag{6}$$

Where $\overline{X}$ is the mean value of X and $E(X)$ its error. It can be also defined $E(X, x_i)$ as the error of X due to the uncertainties of variable x_i. If the error for $x_1, \ldots, x_M$ is statistical and that for $x_{M+1}, \ldots, x_N$ is not, the error of the function X results:

$$E(X) = \sum_{M+1}^{N} \left| E(X, x_i) \right| + \sqrt{\sum_{i=1}^{M} [E(X, x_i)]^2} \tag{7}$$

And:

$$E(X, x_i) = X(x_i + \Delta x_i) - X(x_i) \tag{8}$$

Or in other terms:

$$E(X, x_i) = X(x_i\ f_i) - X(x_i) \tag{9}$$

Where f_i is a basic factor defined as:

$$f_i = 1 + \frac{\Delta x_i}{x_i} \tag{10}$$

And x_i is the mean value of the basic parameters in which uncertainties are postulated.

Basic Selected Parameters For Uncertainties Analysis

The basic or independent parameters selected, in which uncertainties are postulated, and the treatment used to combine them to calculate wall and ONB temperatures, CHF, PRD and their respective ratio errors, are shown in Table 6. The uncertainties are expressed in terms of basic factors. It is important to mention that parameters like flow area or hydraulic diameter are considered as derived variables although in some studies they are used as basic parameters.

Fuel fabrication		**Uncertainty treatment**	**Factor**
a_m	Fuel width	statistical	fa_m
e_m	Fuel thickness	statistical	fe_m
L_m	Fuel length	statistical	fL_m
a_{ch}	Channel width	statistical	fa_{ch}
e_{ch}	Channel thickness	statistical	fe_{ch}
U_{pl}	Plate uranium loading	statistical	fU_{pl}
U_{het}	Plate uranium heterogeneity	statistical	fU_{het}

Operational Measurement			
T_i	Core inlet temperature	systematic	fT_i
Q_{core}	Core volumetric flow	systematic	fQcore
P	Reactor Power	systematic	fP
Correlation			
H_{tc}	Heat transfer correlation	systematic	fH_{tc}
ΔT_{ONB}	ONB correlation	systematic	$f\Delta T_{ONB}$
q''_{CHF}	CHF correlation	systematic	fq''_{CHF}
P_{RD}	Redistribution power	systematic	fP_{RD}
Other: modeling			
Q_o	Core volumetric flow	statistical	fQ_o

Table 6 - Independent parameters in which uncertainties are postulated

Uncertainties in fluid properties are neglected as well as the influence of the variations in atmospheric and hydrostatic pressures on core outlet pressure. If uncertainties in velocity due to form losses (statistical variation in fuel plate inlet geometry) are not considered explicitly, they can be included by a global statistical error in core flow.

Finally it is important to mention that variations of power, core inlet temperature and flow rate inside their respective operational bands are not treated as uncertainties. Therefore when safety-related variables errors are quantified, these operational parameters must be assumed in the worst case values.

Comments and Conclusions

A bibliographic critical review of different methodologies used for uncertainty evaluation in core safety related parameters, was done. Different parameters, where uncertainties are usually considered, were presented and discussed, as well as their intrinsic nature regarding the way the uncertainty combination should be done. It was observed that there is not a unique criterion, to select neither the basic parameters nor the methodologies to combine them. A thorough analysis is required in each case to understand the nature of the variables, their uncertainty propagation and combination in order to perform a coherent study. For example, the use of a factor based method demands a careful application in order to avoid considering uncertainties where they are not present, in particular different factors must be considered for heat flux depending if it is used in wall temperature or flow instability calculations. In the first case local uncertainties should be included while in the second only global effects are relevant. Another usual problem is to over estimate the uncertainty weight in a product where different factors consider the same uncertainty. These kinds of errors are intrinsically eliminated in the direct propagation of uncertainties.

The more thorough the methodology adopted for safety related parameters uncertainties quantification is, the more relaxed correspondent safety margins goals values could be.

A combined statistical method with direct propagation of uncertainties and a set of basic parameters, where uncertainties should be considered, is proposed to calculate wall and ONB temperatures, CHF, PRD and their respective ratio errors.

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